AN ELEGANT MATHEMATICAL SOLUTION FOR THE VENUS TABLET OF AMMISADUQA

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Contents

Background

It is most convenient for me to write this document in the first person as it follows the thought process that I made in arriving at the solution I present.

I was made aware of the Venus Tablet of Ammisaduqa by the then President of the Astronomical Society of Edinburgh (ASE) in 2014, Ken Thomas. It was known to Ken that since retirement I had been developing a very high precision software suite for the visualisation of the celestial sphere for a date range of 2000 BC to 4000 AD, a range that encompasses the emerging potential dates of Ammisaduqa's reign in Babylon. My immediate reaction was that this should be solvable by the use of mathematics alone, applied to the observations and the accurate determination of the positions of the Sun, Moon and Venus with respect to the Earth and as viewed from Babylon.

I worked on the Venus Tablet project for about a year at that time and made a small presentation to the ASE in 2015. I started work again in early 2021 as the ASE was now doing a number of Zoom meetings and I offered to re-visit this project and provide, this time, a Zoom talk. I also delivered this talk to the U3A Astronomy Section in Edinburgh.

It is subsequent to these talks that I have now settled on a mathematical path to a solution which I am publishing in this document.

What this solution is

I am a retired Mathematical Physicist by trade who had a career in developing high precision software suites. For example, I developed a software suite for the treatment of Cancer using High Energy X-rays – so called Radiotherapy. This suite was used worldwide and in the 1990s was the second most widely used suite for the treatment of Cancer.

Following this I was involved in high accuracy navigation of jet aircraft. This required the most detailed analysis of the mathematics of the required software to ensure the highest accuracy – at a time before the now well-known GPS System was made available.

Hence the 'solution' is based on 'Mathematics alone' and not on any specific knowledge of the history of Babylon.

What this solution is not

This solution has only two relationships with Babylon –

1. Where a latitude and longitude are required for the algorithms the latitude and longitude of Babylon have been used, and

2. The development of a Lunar First Crescent based Calendar, as used in Babylon at the time the Venus observations were recorded. This calendar is 'numerical only' and does not have any month names associated with it. The month names add nothing to the mathematics that is to be performed.

I am not an archaeologist and no other information regarding Babylon has been considered in this solution which is based solely on Celestial Dynamics and Mathematical Analysis.

Hence this is not an archaeological document, as such.

A note about Date Formats

In this document you will see dates sometimes displayed as 01/01/-1650 or 01/01/1651 BC. The first is a 'Mathematical' date format and the second is based on the regular BC/ AD nomenclature.

The two differ in that there is a 'mathematical' year of 0 (zero) but this does not exist in the BC/ AD format.

Here is the explanation giving a short sequence including 1 AD –

BC/ AD years   3 BC, 2 BC, 1 BC, 1AD, 2 AD …   and here is the equivalent mathematical sequence –

'Mathematical' -2, -1, 0, 1, 2 …

Note that 1 BC is the year 0, and 2 BC is the year -1. So, as written at the head of this paragraph, 1651 BC is the year -1650. All 'BC' years are 1 different from the mathematical years.

I use both formats in different parts of this document – predominantly mathematical ones in the analysis paragraphs and 'BC' in the results.

Astronomical Algorithms

I want to turn, now, to the celestial sphere algorithms that I am using and the accuracy with which they operate.

I chose the book 'Astronomical Algorithms' by Jean Meeus as the reference work in developing my celestial software. I used the Second Edition, published in 1998 and I started on this project around 2011.

The Planets

For the positioning of the Planets, Meeus introduces a theory of P. Bretagnon initially published as VSOP82 but re-published later as VSOP87 in a form that is more amenable to be represented in software, especially in the form of VSOP87D which Meeus describes.

Meeus provides only a truncated series of parameters for the theory, but I implemented the full series which has been made available, in a suitable form, by Jay Tanner whose website can be found here - https://neoprogrammics.com Hence my planetary positioning algorithm is the **best that can be implemented according to this theory**.

The Moon

For the positioning of the Moon, I have used the parameters straight from Meeus' book. They are related to a Chapront theory ELP 2000 updated later as ELP 2000-82B as quoted in Meeus' book.

I experimented with Chapront's Lunar Tables from 4000 BC to 8000 AD – contained in a separate, but related, book - but did not find that they added anything significant to the accuracy of the tables in

Meeus' book, there being a high dependence on ΔT for both cases. ΔT is a correction used to adjust the speeding up and slowing down of the spinning of the Earth on its axis and it is prone to unknown errors.

I calibrated my implementation of these algorithms using tables of Lunar Eclipses and Venus Transits provided by Fred Espenak and Jean Meeus on the NASA website.

Bearing in mind that the preferred dates for Ammisaduqa's accession to the throne lie in a range of 1650BC + or – 100 years. I checked my calculations for the position of the Moon and Venus at three points in this range – 1550BC, 1650BC and 1750BC – or nearest to those dates, and I chose Total Lunar Eclipses as the best way of ensuring that my software was positioning the Moon correctly with respect to the Sun.

Sighting of the First Crescent

Having done some internet searching on the subject I decided to use the 'Yallop Algorithm' for the first sighting of the new crescent moon. It can be obtained from this web address - https://astro.ukho.gov.uk/download/NAOTN69.pdf

I implemented this algorithm in full and applied it to each New Moon occurrence in order to obtain the first day of the new month in Babylon. I used the latitude and longitude of Babylon in all the calculations.

I assume that this algorithm yields 'mathematically correct' solutions for the required dates but am certain that it cannot be more than one day incorrect, depending on weather conditions at Babylon and the eyesight of the observer. The maximum time for first crescent sighting after conjunction that this algorithm gives in the preferred solution space for the Tablet is 74.6 hours – just over 3 days. If incorrect at this level I would expect the right answer to be 2 days, rather than 4. The minimum time is 25.5 hours – 1 day.

Accuracy of positioning the Moon

Table 1 shows my results for the accuracy of Lunar positioning using Lunar eclipses.

The reference for Espenak and Meeus is their FIVE MILLENNIUM CANON OF LUNAR ECLIPSES: -1999 TO +3000 found here - https://eclipse.gsfc.nasa.gov/SEpubs/5MCLE.html .

Table 1

#	Date	Type	Time E&M	Time calc'd	Difference
1	13/08/-1751	Total	02:35	03:10	0:35
2	12/03/-1650	Total	18:42	20:36	1:54
3	28/09/-1549	Total	19:33	22:41	3:08

The table shows the 3 chosen eclipses. The date of each is in column 2 and the date was the same in my calculation of the eclipse as was that of Espenak and Meeus. My software also predicted a total eclipse.

My calculations only differ from those of Espenak and Meeus in the actual time of the eclipse and the differences are in column 6.

On ‘average’ my calculation would appear to be about 2 hours different.

How does this affect my analysis of the Venus Tablet Observations?

It could result in my dates of First Crescent Moon being incorrect in about 10% of cases, but my calculations would not be incorrect by more than one day at the most, provided the algorithm that I am using to predict the sighting of the First Crescent is accurate. Unfortunately, we are not in a position to fully check this out, but at the most, it also would not be more than one day out.

So, if the conditions for the sighting of the First Crescent in Babylon at the required dates were perfect and the observer had ‘perfect’ eyesight, my first day of the month calculation would not be more than 2 days different – one for the Yallop Algorithm and one for the positioning of the Moon. This is important in the comparison of the days of the month in which Venus becomes invisible and when it subsequently becomes visible again.

Accuracy of positioning Venus

Table 2 shows my results for the Venus Transits.

The reference for Espenak is the SIX MILLENNIUM CATALOG OF VENUS TRANSITS: 2000 BCE TO 4000 CE which can be found here - https://eclipse.gsfc.nasa.gov/transit/catalog/VenusCatalog.html

Table 2

Date	Esp-time	My-time	Time Diff	Esp-sep’n	My-sep’n	Sep’n Diff
18/11/-1755	14:17	13:59	$0^{hr}18^{m}$	7.9’	2.9’	5.0’
23/05/-1649	02:32	02:05	$0^{hr}27^{m}$	11.6’	4.6’	7.0’
20/05/-1641	19:48	19:21	$0^{hr}27^{m}$	7.7’	3.0’	4.7’
20/11/-1520	01:17	00:53	$0^{hr}24^{m}$	14.5’	5.3’	9.2’

In this particular case there are two parameters to check. One is the time of the closest approach of Venus to the centre of the Sun and the other is the closest distance of Venus to the centre of the Sun.

As can be seen my differences in time lie between ¼ hour and ½ hour, and my separation differences are less than 10 minutes of arc – about 1/6th of a degree.

The size of the distance, at 1/6th of a degree ‘could’ have an effect on the days of invisibility and visibility at Superior Conjunctions where Venus moves about ¼ of a degree with respect to the Sun each day.

But at Inferior Conjunctions that movement with respect to the Sun is about 1.6 degrees per day, and it is very likely that my errors would result in only one day of difference in the invisibility and visibility days in a very limited number of cases.

I believe that positioning errors for Venus have no serious effect on my analyses, where I limit those analyses to Inferior Conjunctions only.

Selection of Observations for Analysis

Table 3 shows the original Tablet Observations, as 'corrected' by previous researchers where the observation details had been clearly incorrectly copied from the original clay tablet, which is no longer in existence.

Table 3

#	Year	Month Invis	Name	Day Invis	Month Vis	Name	Day Vis	Days Invis
0	1	XI	Shabatu	15	XI	Shabatu	18	3
1	2	VIII	Arahsamu	11	X	Tebetu	19	2m 7d
2	3	VI	Ululu	23	VII	Tashritu	13	20
3	4	IV	Dumuzi	2	VI	Ululu	3	2m 1d
4	5	II	Ayar	2	II	Ayar	18	15
5	5	IX	Kislimu	12	XI	Shabatu	16	2m 4d
6	6	VIII	Arahsamu	28	IX	Kislimu	1	3
7	7	V	Abu	21	VIII	Arahsamu	2	2m 11d
8	8	IV	Dumuzi	25	V	Abu	2	7
9	8-9	XII	Adar	25	III	Simanu	2	2m 7d
10	9	XII	Adar	11	XII	Adar	15	4
11	10	VIII	Arahsamu	10	X	Tebetu	16	2m 6d
12	11	VI	Ululu	26	VI(2)	Ululu2	7	11
13	12	I	Nisan	9	VI	Ululu	25	5m 16d
14	13	II	Ayar	5	II	Ayar	12	7
15	13	IX	Kislimu	20	XI	Shabatu	21	2m
16	14	VII	Tashritu	10	VIII	Arahsamu	26	1m 16d
17	15	V	Abu	20	VIII	Arahsamu	5	2m 15d
18	16	IV	Dumuzi	5	IV	Dumuzi	20	15
19	16-17	XII	Adar	25	III	Simanu	2	3m 9d
20	17	XII	Adar	11	XII	Adar	15	4
21	18	0	0	0	0	0	0	0
22	19	VI(2)	Ululu2	1	VI(2)	Ululu2	17	15
23	20	III	Simanu	25	VI	Ululu	24	2m 6d
24	21	I	Nisan	27	II	Ayar	3	7
25	21	X	Tebetu	28	XII	Adar	28	2m

Note the sequence numbers in the first column. They start at 0 [Zero] which is common in computer software practice.

Each row of the observations details a pair of observations for the Planet Venus. The first observation is the disappearance of the planet from visibility as it gets close to the Sun and the second is the reappearance of the planet as it moves away from the glare of the Sun.

The moment that Venus is aligned with the Sun is called a Conjunction. Such a conjunction is Superior when Venus is on the far side of the Sun and is called Inferior when Venus is between the Earth and the Sun.

Conjunctions occur in sequences of an Inferior Conjunction followed by a Superior Conjunction, then back to an Inferior Conjunction, etc.

The first row of the Venus Tablet represents an Inferior Conjunction. Venus is invisible for 3 days only. The second is a Superior Conjunction with Venus being invisible for over 2 months – about 67 days in all.

The latter reflects the slow movement of Venus with respect to the Sun at Superior Conjunctions and the former the fast movement at Inferior Conjunctions. **It means that Inferior Conjunctions contain more precise details of Venus' motion compared with the Superior Conjunctions**.

I therefore made the decision to exclude Superior Conjunction Observations from all my analyses. The information contained therein is of low precision. I note that de Jong states in his 2013 paper that these observations can easily be 6-7 days in 'error' between computed and observed days. Such an error cannot be accepted against a 3-day Inferior Conjunction pair of observations, or, indeed in the 20 days which is the longest disappearance recorded on the tablet, at an Inferior Conjunction. The inclusion, therefore, of Superior Conjunctions does nothing more than to dilute the precision of the Inferior Conjunction Observations.

Whilst accepting the Inferior Conjunctions for analysis, I excluded that numbered 16 – year 14 - in the table. It is generally believed that this observation may have been affected by dust in the atmosphere from some Volcanic activity. I will deal with this observation later in this document.

So, out of the 13 Inferior Conjunctions, I am initially including only 12 in my analyses.

The observations for analysis

Table 4 shows the information that is to be analysed. Note that all textual information has been deleted. The names of the months have no relationship with any analysis that is based on mathematics alone.

Table 4

#	Year	Day Invis	Day Vis	Days Invis
0	1	15	18	3
2	3	23	13	20
4	5	2	18	15
6	6	28	1	3
8	8	25	2	7
10	9	11	15	4
12	11	26	7	11

14	13	5	12	7
18	16	5	20	15
20	17	11	15	4
22	19	1	17	15
24	21	27	3	7

Note, also, that the observation numbered 16 has also been deleted from this list.

My algorithms cater for the fact that the sequence of named months is not represented in Table 4 as they depend solely on position in the mathematical sequence of First Crescent dates.

A Babylonian Calendar

It goes without saying that having a Babylonian Calendar is a necessity in order to solve this problem. As I note above, the basic software available to me can place the Moon accurately to 'about 2-3 hours'. This is more than enough accuracy to contemplate obtaining a solution which dates the observations on the Venus Tablet.

So, I developed a software application that performed the following –

1) Established the date and time in Babylon of all New Moons from 2000 BC to 1000 BC

2) Applied the Yallop Algorithm to each New Moon date and time to establish the date of the sighting of the first crescent moon – between 1 and 3 days after New Moon – and hence the sequence of months in the Babylonian calendar.

3) Saved all the information in a database for use with the analyses.

This database needs to be created only once as it never changes.

Among other parameters this database contains, for each New Moon, its date and time in Babylon, Lunar Latitude at New Moon, an index which is the number of the month, starting at zero for the first New Moon in the database, the date of the First Crescent, the time of Sunset and Moonset for that day, the angle between the Sun and Moon, the height of the Moon at sunset, and the phase of the Moon at sunset.

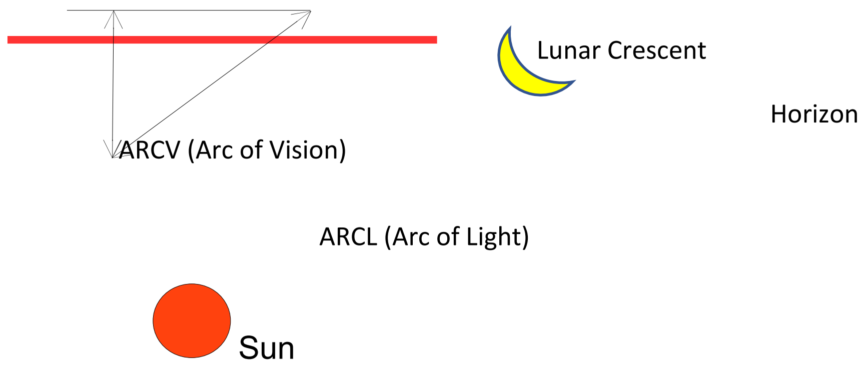
The database is stored in binary form for use by the application software, but also in a textual (.csv) form readable by Microsoft Excel so that it can be manually inspected for any reason.

In summary, though, this database contains 'Babylonian Months' in strict mathematical sequence starting at 0 and incrementing by 1 for each First Crescent Day. There has been no attempt to provide the 'Names' of these months as this is not necessary to solve the problem mathematically.

Arcus Visionis

In order to implement the Yallop Algorithm for the visibility of the first crescent, two parameters need to be taken into account. One is called 'Arcus Visionis' and the other the 'Arc of Light'.

Figure 1



The relevant relationship is shown in Figure 1.

The Arc of Vision, or Arcus Visionis, is the angular height of the crescent Moon above the Sun at sunset. As the Sun sinks below the horizon, the sky begins to darken, and the crescent may be seen.

The Arc of Light is the angular distance of the Moon from the Sun, and this determines how large – and therefore how bright – the Moon is. A larger Arc of Light means that the Moon will be seen at lower heights above the Sun at sunset.

Venus Conjunctions

The next task is to calculate all of the Venus Conjunctions and place them into the Babylonian Calendar that has been produced as in the previous paragraph.

However, unlike the Lunar Calendar which only needs to be calculated once, there is a necessity to have one Conjunction database for each 'Arcus Visionis' that is to be analysed for adherence to the Tablet observations. It is the chosen Arcus Visionis that determines the result that the mathematical analysis will find for the 'First day of invisibility' and the 'First day of visibility' for each row of the tablet observations, and it is these dates that will have to be compared with the Tablet contents in order to solve the problem.

Ultimately, I chose 4 different Arcus Visionis values – 5.0, 5.5, 6.0 and 6.5 – all in degrees, my objective being to test the Tablet Observations against all of these values in order to find a 'best fit'.

I copied all of the Lunar Crescent information that was required into each of the Conjunction databases so that all analyses could be performed on each of the Conjunction databases without further reference to the Lunar Crescent database.

Because all relevant data was stored in the Conjunction databases, all the analyses that I performed were very fast – going through all the solution space in less than 5 seconds in all cases. Mostly the timing was about 1 second.

An 'Elimination' Strategy

All of my initial analyses were performed using an 'Elimination' strategy. I went through the Conjunction databases ruling out all the dates that could not possibly be the solution to the problem.

As a result of this I began to find, as have all researchers, that it was not easy to obtain a 'unique' solution. Problems arise in that 8 years of Earth orbits are almost exactly the same as 13 Venus orbits, giving rise to 'harmonic' solutions separated by multiples of 8 years. Further, 99 average Synodic Periods of the Moon are also almost exactly the same as 8 orbits of the Earth. These cyclical timeframes are a hinderance to arriving at a solution.

But there is one parameter that is helpful and that is the angle of the Venus Orbit to that of the Earth, i.e., the Ecliptic. This is about 3.4 degrees, and it has a significant effect on the number of days of invisibility at Inferior Conjunctions – ranging from 3 to 20 days of invisibility as recorded on the Tablet. This parameter is helpful in finding the right 'blueprint' in the Julian Calendar, for the Tablet Observations.

In view of the fact that previous researchers had concerns regarding the validity of some of the observations – and particularly those beyond year 8 – I tentatively started the elimination process using only 3 sets of observations – those for years 1,3 and 9 – the last one being just outside the 'preferred' good observations.

By using these 3 years I was easily able to bring the good candidate dates down to a list of fewer than 20 entries, with similar results from all of the Arcus Visionis values I had chosen.

At this point I started to look manually at the results of my searches, against the Tablet Observations, and, like other researchers, this was not giving me hope to bring the list down to just one date. I contemplated 'adjusting' the Tablet Observations but decided against such a route. Once the original data has been tampered with there is no way of knowing if one or more of the potential solutions is being advantaged in being selected as the preferred date.

Widening to all the data points

So, I decided to assume that all the observations were equally 'accurate' and that I should seek a solution by looking at all of the data, and not just a 'safe subset'.

I was pleasantly surprised to find that the preferred list of contenders did not change very much when all the [Inferior] observations were taken into account – 12 Observation out of the total of 13 on the Tablet.

However, two observations were always 'errant' in the preferred list. These are year 8 and year 19.

I dealt with these two years separately.

Years 8 and 19

By inspection of the full results of the searches that I made, it seemed that the Tablet entry for year 8 may be 10 days incorrect. The Tablet notes that Venus disappeared on the 25th of the month, yet all the preferred results that I was finding had Venus disappearing 10 or more days earlier than this. Further I could not replicate the 7 days of invisibility.

So, for year 8 I subtracted 10 from the day of invisibility, giving the 15th, and added 10 to the number of days invisible, giving 17. Following this change this particular year gave no more significant problems.

Year 19 was different. There seemed to be no pattern in that year's observations that could easily be interpreted as a 'scribal' error. However, I note that the observations for the previous year, year 18, have been overwritten on the Tablet. Perhaps the extra writing on the Tablet also partially destroyed the year 19 observations?

I decided to delete this year from my analyses.

This left me with 11 years' worth of data points to be analysed – 11 settings and 11 risings.

10 of those years are as per the Tablet generated by previous researchers, and one is for year 8, altered by 10 days.

The two excluded years are year 14 – suspected volcanic intervention – and year 19 – which looks to have been damaged.

The Arc of Light and 'Arcus Visionis' for Venus

Like all researchers are reporting, there were some observations that did not seem to be 'correct' in that they are not in synergy with the main solutions that emerge when using an elimination process to find the correct solution.

In my suite I have software algorithms that allow me to visualise the celestial sphere on my monitor. I enhanced this software to provide me with imagery for both Sunrise and Sunset so that I could clearly see the behaviour of Venus.

I will here provide some limited imagery for the conjunction of 12/10/-1635, a date that affects both of the Middle Chronologies.

The imagery is drawn as it would be viewed from the latitude and longitude of Babylon, but the times are those at Greenwich.

The relevant values are –

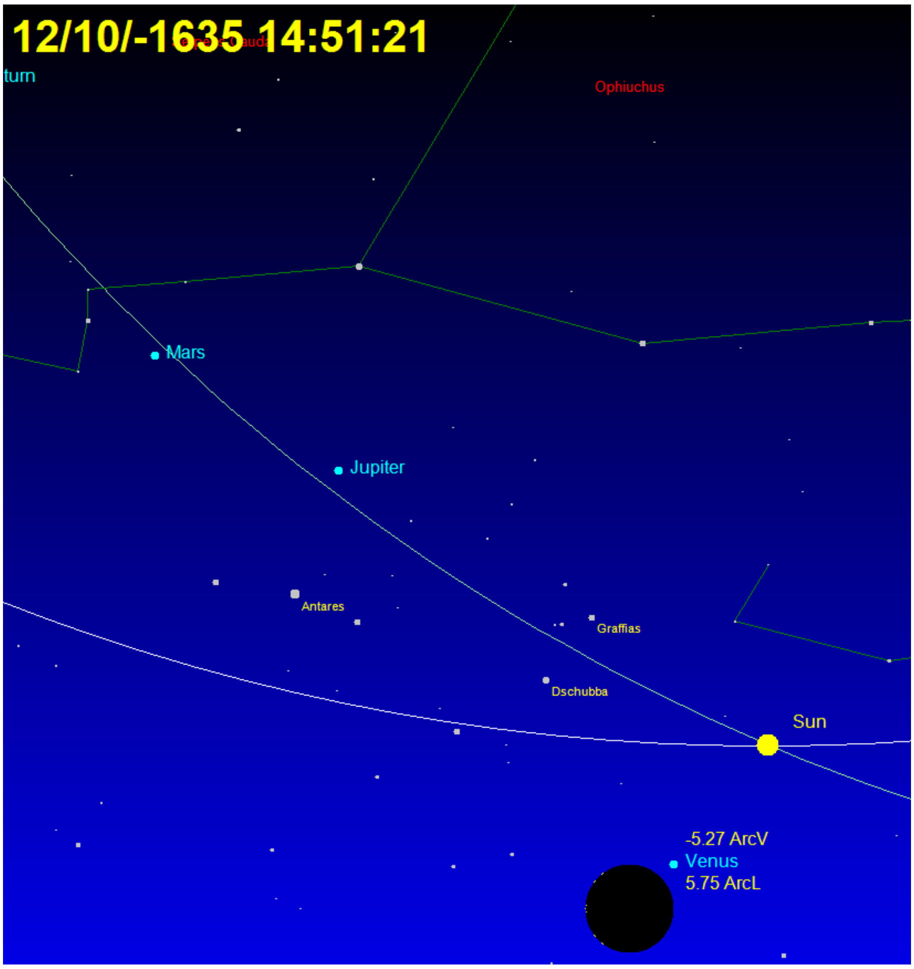
Latitude of Babylon – 32.54 degrees North,

Longitude of Babylon – 44.42 degrees East and

Time difference from Greenwich – +2^{hr}57^{min}40^{sec}

Firstly, here is an image on the day of the Conjunction.

Figure 2



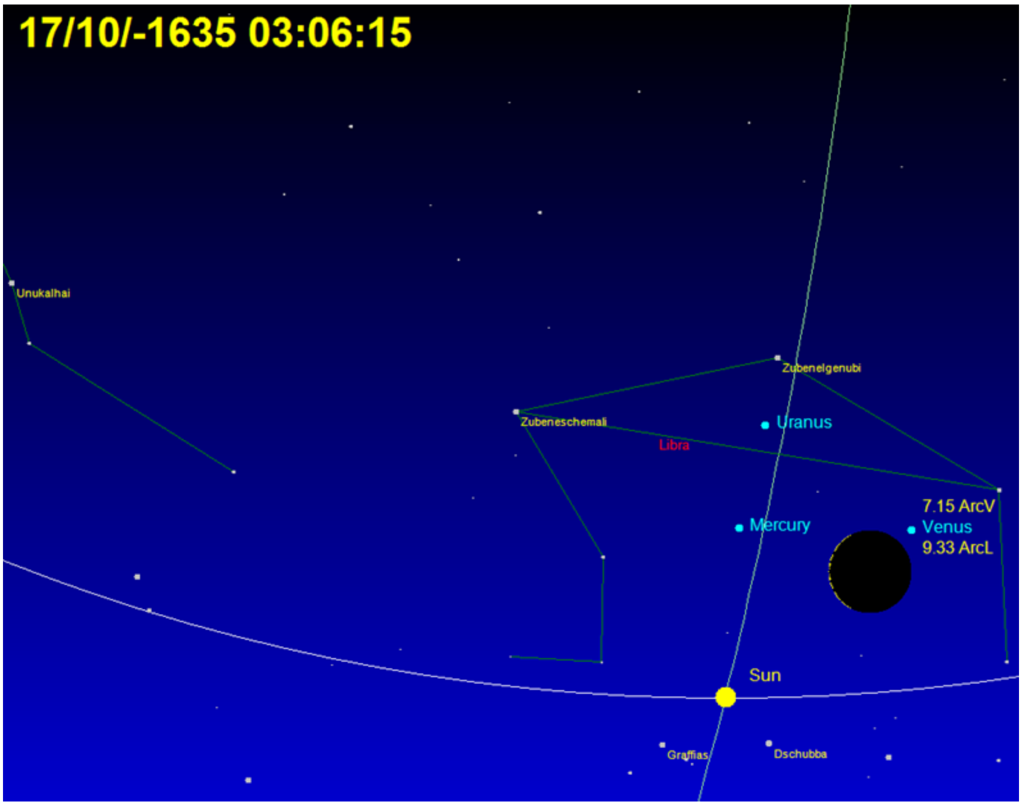
In this image the Horizon is the white curve going through the Sun and the Ecliptic is the green line. We see that the Ecliptic is at an acute angle with respect to the horizon. This is not unusual in the solution space, and, indeed, the angle is sometimes more acute than it is here.

Note that I have arranged for both ArcV [Arcus Visionis – which is the height above the horizon at sunset] and ArcL [Arc of Light - which is the angular distance of Venus from the Sun] to be displayed next to Venus, and I also (dark circle) give the size of the Venus Crescent – on the basis that Venus has no atmosphere.

Previous researchers have always relied on the fact that unlike the Moon, the visibility of Venus is linked to ArcV only, and not to ArcL.

Next, I reproduce the image 5 days later, and at Sunrise.

Figure 3

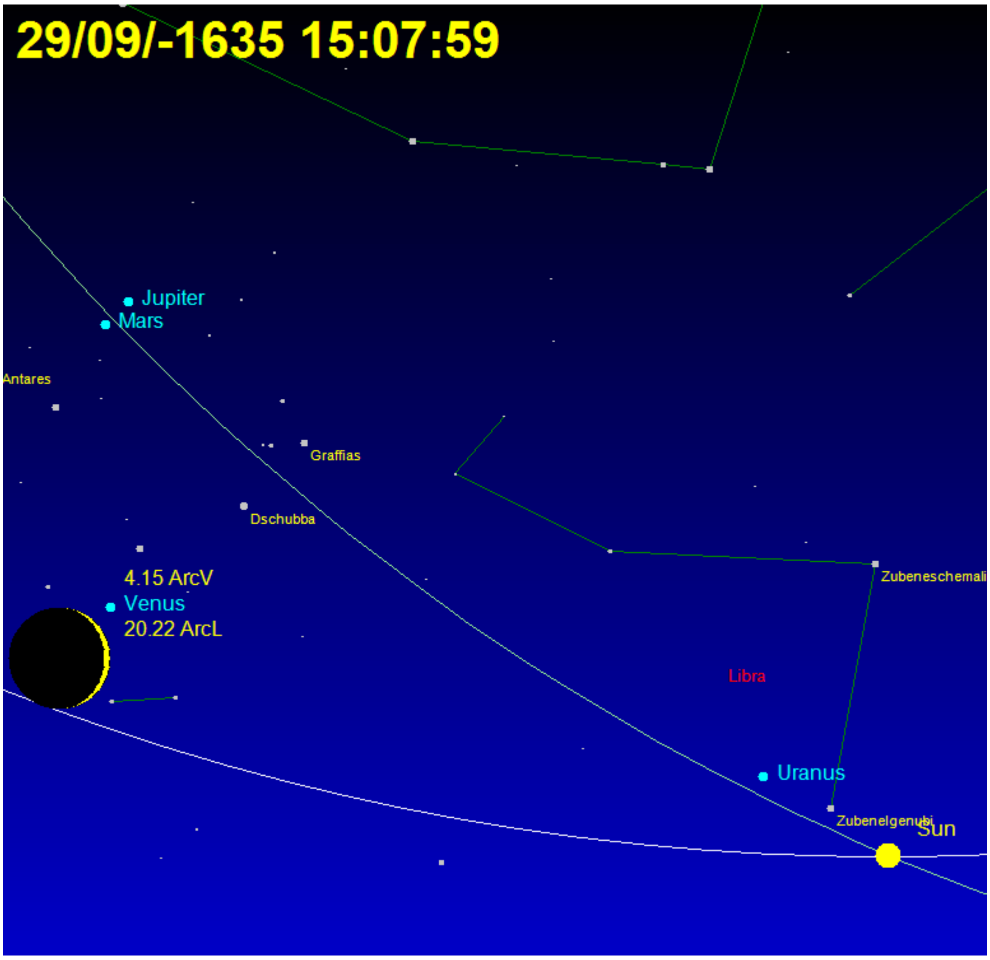


We can see that Venus has risen to 7.15 degrees above the horizon and would be clearly visible. Note that at sunrise, the Ecliptic is almost perpendicular to the Horizon, and this means that Venus will rise very rapidly.

Also note that ArcL is 9.33 degrees, and the crescent is quite small.

Now going back to the time that Venus set prior to the Conjunction.

Figure 4



Note that in this image – already 13 days prior to Conjunction – Venus, 4.15 degrees above the horizon, will be deemed to be extinguished when looking at ArcV's of 5 degrees and higher. Indeed, I could have taken the date further back and still been inside 5 degrees, but Venus would have moved off the side of the image.

However, the point which I want to make, is that ArcL at this moment is 20.22 degrees and that Venus now has quite a visible crescent.

This leads to a question – Would Venus be visible in the sky, at heights above the horizon lower than ArcV if its ArcL is large enough?

I spent some time looking at this problem by inspecting imagery such as represented above but could find no indication in the literature that would give me a steer as to how to formulate a mathematical algorithm to connect ArcV and ArcL for Venus. Further, looking at 'favoured' solutions manually – over all their Inferior Conjunctions – did not lead me to a situation in that I could obtain a 'best fit' to the Tablet contents when the Ecliptic was at a very acute angle to the Horizon.

I have a concern that the four individual ArcV values suggested by de Jong do not consider situations such as this one where Venus is on a 'grazing angle' path with respect to the horizon.

I therefore abandoned this [manual] process and reverted to using 'constant' ArcV values only.

Figure 5

However, before I leave this topic, I present a sequence of images taken in 2004.

We can see that, unlike the Moon, the atmosphere of Venus has an effect on its brightness at Inferior Conjunction and indeed, even when fully backlit the atmosphere presents a non-dark image.

Image Attribution: Statis Kalyvas - VT-2004 programme.

Taking Stock

Although everything I have reported to date in this document was performed with the sole objective of 'solving' the date for the Venus Tablet, like other researchers I found myself unable to recommend any one 'preferred' solution over another one.

Nevertheless, performing all these preliminary analyses was important as it gives an insight into the nature of the observations and therefore a clue as to how to proceed next.

In my original attempts to solve the problem I created a very precise Babylonian calendar using the Yallop Algorithm and 'set it to one side'. I then created a Babylonian calendar from the 25 rows of observations on the Venus Tablet.

Finally, I sought to 'insert' this latter calendar into the high precision one. And I found out the hard way that trying to fit a 'somewhat low precision' calendar into a high precision template does not necessarily lead to a high precision solution.

I therefore decided to change direction. In a new approach I now worked from the mathematically produced high precision Babylonian calendar. And I directly inserted the, presumed high precision, Tablet observations, in order, into this calendar.

This has proved to be a good way of preserving the precision of the calendar and the precision of the observations, leading ultimately to a single, preferred, solution.

In the descriptions to this point I have not described the manner in which I was ranking potential solutions and being able to eliminate non-starters. I was using a 'least squares' process. 'Least Squares' is a mathematical process that is commonly used to extract a graphic trend from a number of data points that have random errors. There is always one 'winner' – the solution with the smallest least squares value.

In seeking a solution to the Venus Tablet, the data points with the 'random' errors are the Tablet observations of 'Day of month that Venus was invisible' and 'Day of month that it became visible again'. These can be compared, using a least squares procedure, with the same variables that have been calculated with high precision using a proposed ArcV below which Venus cannot be seen by eye.

Each term in the process is $(\text{Tablet-Day} – \text{Computed-Day})^2$ and these terms are added up to give a 'Figure of Merit' for each of the solutions under consideration. The smallest one is the one in which 'more of the data points are closer to the computed points than all of the other potential solutions'.

It is a process in which ALL the data points (observations) are given EQUAL status and no one observation is singled out for special treatment. **This is a very important feature of the process**.

So, I programmed the algorithm and then stepped back, forgetting everything known about the observations, and **letting the mathematics freely go to the best place**.

The Least Squares implementation

In the above description the terms to be added have been stated to be the squared differences between dates on the Tablet and computed dates for a particular ArcV.

However, as noted earlier I believe that the precise Babylonian calendar, calculated by mathematics alone, can easily be up to two days different from the one recorded on the Tablet. So, I adjusted the terms in my Least Squares implementation to take 2 days off before squaring them for the process.

i.e., Each term is $(|\text{observed} – \text{calculated}| - 2)^2$ where if the initial difference is zero or one the term to be squared is set to zero.

As I noted earlier in this document, I started with 12 of the 13 Inferior Conjunctions in the process but in early analyses dropped this to 11. This gives a total of 22 terms to be added to a single Figure of Merit.

As the number of days of invisibility is the difference between the setting and rising dates involved in the least squares, it is not included in the calculation. It is the setting and rising dates of the observations that have to be inserted into the mathematically precise calendar. Nevertheless, I calculated the expression for the days of invisibility and inspected it when looking at the results of the process.

Initially I used a solution space of 1500 BC – 1800 BC. This yielded an answer.

I then expanded the solution space to 1000 BC – 2000 BC and discovered a new and very interesting 'harmonic' involving the Earth's orbit and that of Venus. This discovery strengthens my belief that the Tablet observations, with the 2 possible exceptions, are genuine observations and that they must all be treated as 'equal in accuracy'.

I report on these results below.

The software interface

As a result of all the early analyses I gradually created an interface that enabled me to rapidly check on results and, indeed, include or exclude any of the observations on the Tablet.

The screen is reproduced below in Figure 6.

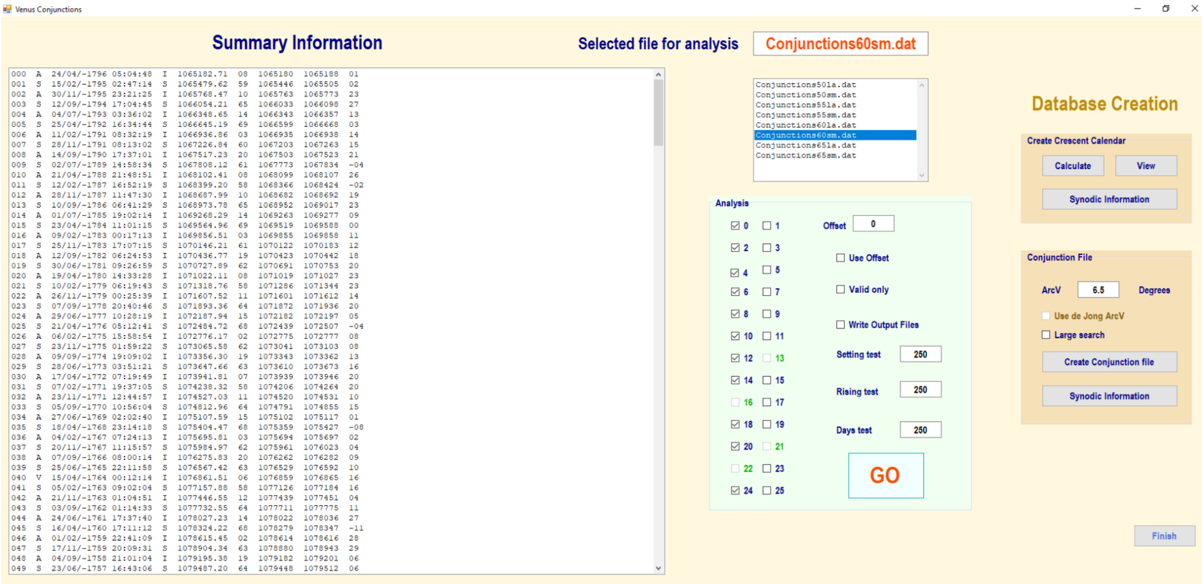
The right-hand side contains two 'Groups' of controls that are used to create the required databases. The top group is for the creation of the precise Babylonian Calendar, and it is a prerequisite to the second group below it that creates the various 'Conjunction' databases which depend on the selected ArcV values.

The Conjunction files are given unique names that include the digits of the ArcV value and whether they are ‘small’ or ‘large’ databases.

I sampled the solution space in two different date ranges. The ‘small’ solution space is from 1500 BC to 1800 BC which includes the dates that are ‘preferred’. The ‘large’ solution space which is used whenever the check box ‘Large Search’ is checked, uses all dates between 1000 BC and 2000 BC.

The List Box above the green ‘Analysis’ group contains a list of all the ArcV files that have been created and one is selected for analysis by double-clicking on its name in the List Box. It is then displayed in the ‘Selected file for analysis’ Text Box and various properties are displayed in the ‘Summary’ List Box.

Figure 6



The ‘Analysis’ screen

Note that in the ‘Analysis’ group there are 26 Check Boxes. These are used to include or exclude any of the Tablet Observations that are desired. It is populated on entry with a selection of Inferior Conjunction Observations [even numbers] but with all the Superior Conjunctions unchecked.

There are 4 checks that cannot be activated. Checks 13 and 16 are the two sets of observations that are almost certainly related to volcanic activity. Check 21 is the one missing Super Conjunction Observation and Check 22 is the one Inferior Conjunction Observation that appears to be corrupted.

An analysis is actioned by clicking the ‘Go’ button.

The ‘small’ analyses take about 1-2 seconds, and the ‘large’ analyses less than 5 seconds. This means that I can rapidly look at any grouping of observations very quickly.

Indeed, once I reached this level of capability in the software, I almost always included all 11 Inferior Conjunction observations with no level of ‘special treatment’ for any of them.

The ‘Offset’ value, its Check Box and the Check Box labelled ‘Valid Only’ are left over from the time I was trying to arrive at a solution by elimination. In early runs, I marked non-starting dates with a code indication that they had been eliminated, so the analysis software subsequently ignored them. But as it takes only 1-2 seconds for a full analysis, I stopped using this feature. The 3 test values – Setting, Rising and Days - were also associated with the elimination strategy.

Finally, I used the 4 unique values of de Jong in respect of the ArcV properties in some early analyses, but settled on single values once the full software had been developed – performing the final analyses with ArcV's of 5.0, 5.5, 6.0 and 6.5 degrees. See also my previous comments on when Venus approaches or leaves the horizon at a 'grazing angle'. There are 8 databases. Four for the 'small' analyses, and four for the 'large' analyses. I chose not to use the de Jong unique values for ArcV both as they had not led me any closer to a solution than when I had been using a constant value and in consideration that they do not encompass that it may be possible to see Venus below the ArcV level if ArcL is large enough.

1500 BC – 1800 BC

I ran the Least Squares procedure for all four chosen ArcV's. The best solution in all four cases was 1645BC. [Note that this is the date of the first entry on the Tablet and not necessarily the first year of Ammisaduqa's reign, usually quoted one year earlier at 1646 BC].

Table 5 summarises the Least Squares results for 1645 BC.

Table 5

ArcV in Degrees	Σ Setting DOM	Σ Rising DOM	Σ Days Invisible	Σ Setting + Rising
6.5	81	41	62	122
6.0	63	40	38	103
5.5	60	43	17	103
5.0	60	52	20	112

Where 'DOM' is 'day of Month' and Σ means 'Sum of Squares with 2 days subtracted' [and 3 for the Days Invisible which is for 'show' only and is not part of the decision process].

The two central ArcV's are lower than the others and I chose 6.0 degrees as the ArcV to use for all further analyses.

Table 6 gives the top 11 Least Squares results using an ArcV of 6.0 degrees. The entries are sorted on the values of the last column.

Table 6

Date	Σ Setting DOM	Σ Rising DOM	Σ Days Invisible	Σ Setting + Rising
11/03/1645 BC	63	40	38	103
24/02/1589 BC	57	94	54	151
21/02/1581 BC	103	79	49	182
29/03/1709 BC	139	79	42	218
28/01/1744 BC	118	132	130	250
13/03/1653 BC	102	187	41	289
27/03/1701 BC	210	96	34	306
15/04/1765 BC	238	70	58	308
07/02/1533 BC	135	193	86	328
09/03/1637 BC	271	166	39	437

25/04/1554 BC	327	132	96	459

The four favoured dates are 1701 BC [High Chronology], 1645 BC [Upper Middle Chronology], 1637 BC [Lower Middle Chronology] and 1581 BC [Low Chronology].

We can see that the High Chronology comes 7th in the rankings, the Lower Middle Chronology 10th and the Low Chronology 3rd. However, an 'Upper' Low Chronology comes 2nd.

Given the harmonic nature of the orbits, it is not surprising that there is not a big gap between the best fit and some of the others.

Interestingly, if one further observation line is deleted from the procedure, that for year 13, 1645 BC looks better against the other contenders. See Table 7 below.

Table 7

Date	Σ Setting DOM	Σ Rising DOM	Σ Days Invisible	Σ Setting + Rising
11/03/1645 BC	14	39	37	53
21/02/1581 BC	54	54	49	108
24/02/1589 BC	53	94	54	147
29/03/1709 BC	90	79	26	169
27/03/1701 BC	110	80	25	190
28/01/1744 BC	102	116	130	218
15/04/1765 BC	157	66	42	223
25/01/1736 BC	105	131	125	236
09/03/1637 BC	150	130	38	280
13/03/1653 BC	98	187	37	285
07/02/1533 BC	135	193	86	328
11/02/1792 BC	143	195	57	338

However, I reverted to using all 11 observations after noting this particular feature of 1645 BC. I note that the Low Chronology still comes second, albeit the upper and lower dates reverse.

1000 BC – 2000 BC

I will report here the results of the Least Squares procedure, using all 11 observations and an ArcV of 6.0 degrees.

Firstly, Table 8 shows the top 50 results of the extended search. I elected to list 50 as the date in 11th place in the smaller analysis is ranked 48th in this large analysis.

Table 8

Rank	Date	Σ Set DOM	Σ Rise DOM	Σ Days	Σ Set + Rise
1	04/03/1864 BC	33	50	38	83

2	11/03/1645 BC	63	40	38	103
3	03/03/1370 BC	47	65	53	112
4	10/03/1151 BC	70	47	39	117
5	22/02/1095 BC	50	68	49	118
6	12/02/1306 BC	57	77	50	134
7	24/02/1589 BC	57	94	54	151
8	13/02/1800 BC	77	90	68	167
9	28/03/1215 BC	104	65	22	169
10	05/02/1525 BC	73	98	86	171
11	21/02/1581 BC	103	79	49	182
12	21/03/1434 BC	83	108	33	191
13	27/01/1250 BC	67	126	139	193
14	20/03/1920 BC	136	59	38	195
15	19/02/1087 BC	90	107	62	197
16	22/03/1928 BC	89	109	29	198
17	19/03/1426 BC	122	76	33	198
18	29/03/1709 BC	139	79	42	218
19	14/02/1314 BC	92	130	61	222
20	12/03/1159 BC	96	147	38	243
21	02/03/1856 BC	144	106	41	250
22	28/01/1744 BC	118	132	130	250
23	07/04/1490 BC	193	64	36	257
24	16/02/1808 BC	117	152	54	269
25	07/04/1984 BC	205	76	41	281
26	13/03/1653 BC	102	187	41	289
27	14/04/1271 BC	232	63	42	295
28	26/03/1207 BC	192	103	34	295
29	01/03/1362 BC	176	129	40	305
30	27/03/1701 BC	210	96	34	306
31	15/04/1765 BC	238	70	58	308
32	07/02/1533 BC	135	193	86	328
33	20/01/1963 BC	175	172	147	347
34	19/01/1469 BC	175	177	181	352
35	18/01/1955 BC	173	180	187	353
36	25/01/1736 BC	154	212	125	366
37	09/01/1680 BC	227	161	238	388
38	05/03/1378 BC	169	222	37	391
39	07/01/1186 BC	222	177	263	399
40	01/01/1899 BC	236	170	292	406
41	17/01/1461 BC	219	197	202	416
42	24/01/1242 BC	197	224	155	421
43	05/04/1976 BC	320	107	36	427
44	09/03/1637 BC	271	166	39	437
45	04/04/1482 BC	311	132	23	443
46	03/02/1517 BC	198	252	99	450
47	08/03/1143 BC	280	174	52	454

48	25/04/1554 BC	327	132	96	459
49	16/04/1279 BC	269	192	59	461
50	31/12/1406 BC	298	165	287	463

In this larger search, the best fit in the smaller search only ranks second, and 1864 BC is now the best fit. This solution is not related to 1645 BC by any factor of 8. Indeed, it is 219 years away from 1645 BC.

I looked at the properties of this 219 years' difference by seeing if there is a resonance between the orbits of the Earth and Venus.

Using NASA lengths in days for the Tropical year in each case yielded a resonance point.

Earth – 365.242 days X 219 orbits = 79,988.0 days and

Venus – 224.695 days X 356 orbits = 79,991.4 days.

The difference is only 3.4 days. And going forwards in time, there should be a resonance in the years 1426 BC and 1207 BC. The former is ranked 17th in the results and the latter 28th. Virtually all of the preferred results are related to the two series that differ by 219, with the resonances being commonly 8, 56 and 64 years apart.

Archaeologists may want to consider the repercussions of selecting 1864 BC as the date of the first observation on the Venus Tablet.

The fact that the Least Squares procedure uncovered this resonance pattern gives me more confidence that this process has found the best fit to the Venus Tablet observations and also that the observations themselves, as a set, are very consistent. **Remember precise observations are being mathematically compared with a precise calendar**.

Volcanic Activity

Researchers have noted that at least one of the observation pairs, namely year 12, appears to be representative of the footprint of volcanic activity and I note, that in addition to this the observation pair for year 14, although shorter in elapsed time, may well also be a volcanic footprint.

In a previous talk that I gave, I followed the route of associating these two footprints with the Santorini volcano in the Mediterranean Sea. I also noted that some researchers were attempting to associate this volcano with the Biblical Exodus from Egypt.

I have researched these points more deeply and the up-to-date view seems to be that the Exodus was several hundred years after the date, 1650BC, often associated with Santorini. I therefore withdraw the notion that Santorini was associated with the Exodus and state that 'more information is necessary' for that to be confirmed or denied.

I have found a fairly recent paper, published in 2019, regarding probable volcanic activity around the time that my Least Squares process has identified for the date of the Venus Tablet. This paper can be found here - https://www.cambridge.org/core/journals/antiquity/article/absolute-treering-dates-for-the-late-

bronze-age-eruptions-of-aniakchak-and-thera-in-light-of-a-proposed-revision-of-icecore-chronologies/0E06053BFD90C1EA58ED8822814DC6F8#article and it was written by Jonny McAneney and Mike Baillie.

They have found evidence of volcanic activity around this time that is not related to Santorini. Their hypothesis is that the volcano in question was Aniakchak volcano in Alaska. Their thesis is based on Ice Core and Tree Ring data that matches the foot print of Aniakchak, and not Santorini. Many other researchers are working on a very similar thesis to them.

I am not in a position of confirming or denying this thesis. I can only report, here, on the dates that are associated with my solution for the dating of the Venus Tablet.

Relevant dates

Given that the first year of information on the Tablet is 1645 BC, the appearance of the first, and larger, volcanic footprint would have occurred between late 1636 BC and early 1635 BC. The second, smaller footprint, would have occurred between early 1633 BC and late 1633 BC.

McAneney and Baillie have hypothesised that Santorini may have exploded at or near the eruption of Aniakchak and thus these two volcanoes could be associated with the two dates that I note, in either order – perhaps dependent on Ice Core and Tree Ring data.

I cannot make this selection, as I am only reporting a mathematical resolution of these dates, tied only to the Venus Tablet.

There is also an observation pair in year 19 that has a larger than normal Venus disappearance but like other researchers I believe this observation pair to contain scribal errors.

Date Range

With 1645 BC being the first date on the Tablet, the last date is 1625 BC. Outside of this date range, I can make no comments on the dates of either volcano. As the Lower Chronology always seems to come second in the Least Squares process, perhaps there is a necessity to see if there are any volcanic activity dates that can be associated with that solution.

Summary

It is my opinion that several copies of the Venus Tablet of Ammisaduqa have survived, and were inscribed with the words 'The year of the Golden Throne', because the Babylonians knew that there were two pairs of observations that were different from anything they had seen before and that they did not understand them.

Those observations relate to the 12th and 14th years of the Tablet. I'm sure that this tablet was handed down in the hope that one day these observations could be explained.

These observations would tend to indicate particles in the atmosphere that were interfering with the light from Venus. Such particles could be volcanic in nature, or even the result of a collision by the Earth with either a Comet or a Meteor. The most likely explanation is that they are volcanic in nature and McAneney and Baillie have published papers relating to potential volcanic activity 'in the region of the dates represented on the Tablet' where the date of the first observation is in 1645 BC. Previously de Jong has suggested that the date of the Tablet is 8 years later – starting in 1637 BC. His solution seems to fit the McAneney and Baillie thesis very well, but it does not show up as well as 1645 BC when it is subjected to a solely mathematical analysis.

I note that both of the dates 1645 BC and 1637 BC refer to the 'Middle Chronology'. Coming 'second' in the Least Squares analysis are the two dates 1581 BC and 1589 BC which represent the Low Chronology. The dates for the High Chronology are further back.

I note, again, that this research, leading to 1645 BC, was performed 'strictly mathematically' on precise observations – we have to accept these on 'trust' – and a precise Babylonian Calendar, generated by software. In the latter case we have to rely on modern theories regarding the historical behaviour of objects in the Solar System, and also the implementation of the Yallop Algorithm for detecting the 'First Crescent Moon'.

For me, the association with volcanic activity is a compelling argument and the solutions of 1645 BC and 1637 BC are sympathetic with the volcanic activity described by McAneney and Baillie. My own preference, at this time, is 1645 BC.